Published by Institute of Physics Publishing for SISSA
Received: *January 11, 2006*
Revised: *February 22, 2006*
Accepted: *March 10, 2006*
Published: *March 23, 2006*

Production of neutral fermion in linear magnetic field through Pauli interaction

Hyun Kyu Lee and Yongsung Yoon

Department of Physics, Hanyang University
Seoul 133-791, Korea
E-mail: hyunkyulee@hanyang.ac.kr, cem@hanyang.ac.kr

Abstract: We calculate the production rate of neutral fermions in linear magnetic fields through the Pauli interaction. It is found that the production rate is an exponentially decreasing function with respect to the inverse of the magnetic field gradient, which shows the non-perturbative characteristics analogous to the Schwinger process. It turns out that the production rate density depends on both the gradient and the strength of magnetic fields in 3+1 dimension. It is quite different from the result in 2+1 dimension, where the production rate depends only on the gradient of the magnetic fields, not on the strength of the magnetic fields. It is also found that the production of neutral fermions through the Pauli interaction is a magnetic effect whereas the production of charged particles through minimal coupling is an electric effect.

Keywords: Electromagnetic Processes and Properties, Nonperturbative Effects.

Contents

1. Introduction

It is well known that the interaction of charged spin-1/2 fermions with the electromagnetic field is described by the minimal coupling in the form of Dirac equation. Pauli [1] suggested a non-minimal coupling of spin 1/2-particle with electromagnetic fields, which can be interpreted as an effective interaction of fermion to describe the anomalous magnetic moment of fermions [2]. Pauli interaction is particularly interesting for describing the interaction of neutral particle with electromagnetic field provided it has a non-vanishing magnetic dipole moment [3]. One of the immediate possibilities [4] is the electromagnetic interaction(Pauli interaction) of neutrinos, which are recently confirmed to have a non-zero mass with mixing [5]. The presence of the magnetic dipole moment implies that neutrino can directly couple to the electromagnetic field, which leads to a variety of new processes [6, 7].

One of the interesting phenomena with the strong electromagnetic field configuration is the pair creation of particles. The well known example is the Schwinger process with the minimal coupling, in which charged particles are created in pairs [8] under a strong electric field. However it has been demonstrated that no particle creation is possible under the pure magnetic field configuration even with the spatial inhomogeneity [9]. For a neutral particle with the Pauli interaction, the inhomogeneity of the magnetic field coupled directly to the magnetic dipole moment plays an interesting role analogous to the electric field for a charged particle. The non-zero gradient of the magnetic field can exert a force on a magnetic dipole moment such that the neutral fermion can get an energy out of the magnetic field. Hence it will affects the vacuum structure greatly for the strong enough magnetic field as for the case of charged particles in the strong electric field. It is then interesting to see whether the vacuum production of neutral fermion with a non-zero magnetic moment in an inhomogeneous magnetic field is possible on the analogy of the Schwinger process. Interestingly it has been demonstrated in 2+1 dimension that the magnetic dipole coupled to the field gradient induces pair creations in a vacuum [10]. In this work, we will present a realistic calculation in 3+1 dimension for the pair production rate of neutral fermions with non-vanishing magnetic dipole moment.

JHEP03(2006)078

2. Production rate of neutral fermions through Pauli interaction

The simplest Lagrangian for the neutral fermion, which couples to the external electromagnetic field, was suggested by Pauli long time ago [1]. The Dirac equation with Pauli term is given by

$$\mathcal{L} = \bar{\psi}\left(\not{p} + \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu} - m\right)\psi, \tag{2.1}$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, $g_{\mu\nu} = (+,-,-,-)$. μ in the Pauli term measures the magnitude of the magnetic dipole moment of fermion. Pauli term can be considered as an effective interaction which describes anomalous magnetic moment of fermion or as an effective magnetic moment induced by the bulk fermions in a theory with large extra dimensions [11]. The corresponding Pauli Hamiltonian operator is

$$H = \vec{\alpha}\cdot(\vec{p} - i\mu\beta\vec{E}) + \beta(m - \mu\vec{\sigma}\cdot\vec{B}), \tag{2.2}$$

where $\sigma^i = \frac{1}{2}\epsilon^{ijk}\sigma^{jk}$. If the magnetic field is stronger than the critical field, $B_c \equiv m/\mu$, a negative energy state of $m - |\mu B|$ appears. Thus, we will consider magnetic fields weaker than the critical magnetic field, $B < B_c$.

In general, the effective potential, $V_{\rm eff}(A)$, for a background electromagnetic vector potential, A_μ, can be obtained by integrating out the fermion:

$$i\int d^4x V_{\rm eff}(A[x]) = \int d^4x \langle x|\mathrm{tr}\ln\left\{\left(\not{p} + \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu} - m\right)\frac{1}{\not{p} - m}\right\}|x\rangle, \tag{2.3}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and tr denotes the trace over Dirac algebra. The decay probability of the background magnetic field into the neutral fermions is related to the imaginary part of the effective potential $V_{\rm eff}(A)$,

$$P = 1 - |e^{i\int d^4x V_{\rm eff}(A[x])}|^2 = 1 - e^{-2Im\int d^3x dt V_{\rm eff}(A[x])}. \tag{2.4}$$

That is, the twice of the imaginary part of the effective potential $V_{\rm eff}(A[x])$ is the fermion production rate per unit volume: $w(x) = 2Im(V_{\rm eff}(A[x]))$ for small probabilities.

Using the charge conjugation matrix C:

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T, \quad C\sigma^{\mu\nu}C^{-1} = -\sigma^{T\mu\nu}, \tag{2.5}$$

and the identity [12]

$$\ln\frac{a}{b} = \int_0^\infty \frac{ds}{s}\left(e^{is(b+i\epsilon)} - e^{is(a+i\epsilon)}\right), \tag{2.6}$$

we can write the effective potential $V_{\rm eff}(A[x])$ as follows

$$V_{\rm eff}(A[x]) = \frac{i}{2}\int_0^\infty \frac{ds}{s} e^{-ism^2}\mathrm{tr}\left(\langle x|e^{is(\not{p}+\frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu})^2}|x\rangle - \langle x|e^{isp^2}|x\rangle\right). \tag{2.7}$$

The interaction part of the first term takes the following form ;

$$\left(\not{p} + \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)^2 = p^2 + \frac{\mu^2}{4}(\sigma^{\mu\nu}F_{\mu\nu})^2 + \frac{i\mu}{2}\{\gamma^\alpha, \gamma^\mu\gamma^\nu\}\{p_\alpha, F_{\mu\nu}\} + \frac{i\mu}{2}[\gamma^\alpha, \gamma^\mu\gamma^\nu][p_\alpha, F_{\mu\nu}]. \tag{2.8}$$

Because the last three terms in eq. (2.8) are explicitly dependent on Dirac matrices and space-time in general, one can not expect the decoupling of Dirac algebra as in the minimal coupling [1]. It is the major complication in computing the effective action for Pauli coupling. Interestingly in 2+1 dimension one can see the decoupling is possible due to the special properties of Dirac matrices as shown below. The only relevant indices are $\mu, \nu = 0, 1, 2$ and F_{12} for the field strength. Hence the non-vanishing part in the third term in eq. (2.8) is found to be

$$\{\gamma^\alpha, \gamma^\mu\gamma^\nu\}\{p_\alpha, F_{\mu\nu}\} = 2\{\gamma^0, \gamma^1\gamma^2\}\{p_0, F_{12}\}. \tag{2.9}$$

Since, in 2+1 dimension, $\gamma^1\gamma^2 = -i\gamma^0$, which can be verified with a particular representation, $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^1$, $\gamma^2 = i\sigma^2$, we get

$$\{\gamma^\alpha, \gamma^\mu\gamma^\nu\}\{p_\alpha, F_{\mu\nu}\} = -4i\{p_0, F_{12}\}, \tag{2.10}$$

which is free from Dirac algebra. Whereas the non-vanishing parts in the fourth term are given by

$$[\gamma^\alpha, \gamma^\mu\gamma^\nu][p_\alpha, F_{\mu\nu}] = -2\gamma^2[p_1, F_{12}] + 2\gamma^1[p_2, F_{12}]. \tag{2.11}$$

For a linear field strength, the commutators in the right hand side of eq. (2.11) become constants so that the fourth term commutes with the other terms in eq. (2.8) and we get a similar result as in the minimal coupling [10].

As in 2+1 dimension, we consider a static linear magnetic field configuration with a constant gradient along an orthogonal direction to the magnetic field in 3+1 dimension. We take $\hat{z}$-direction as the magnetic field direction with a constant gradient along $\hat{x}$-direction, $\vec{B} = B(x)\hat{z}$, such that

$$F_{12} = B(x) = B_0 + B'x = B'\widetilde{x}, \qquad \left(\widetilde{x} = x_* + x, \quad x_* = \frac{B_0}{B'}\right), \tag{2.12}$$

where the field gradient B' is a non-zero constant.

For this background magnetic field, with the help of Dirac and Heisenberg algebra, the interaction term in eq. (2.8) can be reduced as follows

$$\left(\not{p} + \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)^2 = -p_1^2 - p_2^2 + \left(\mu B'\widetilde{x} + i\widetilde{\gamma^3}p_0 + i\widetilde{\gamma^0}p_3\right)^2 - \mu B'\gamma^2 , \tag{2.13}$$

where $\widetilde{\gamma^3} \equiv \gamma^0\gamma^1\gamma^2$ and $\widetilde{\gamma^0} \equiv \gamma^1\gamma^2\gamma^3$. Since $[\widetilde{\gamma^3}, \gamma^2] = 0$ and $[\widetilde{\gamma^0}, \gamma^2] = 0$, $\widetilde{\gamma^3}p_0 + \widetilde{\gamma^0}p_3$ commutes with the other terms in eq. (2.13) and the last term can be factorized. Hence, we can make use of the following identity [12]

$$\begin{aligned} &p_1^2 + p_2^2 - \left(\mu B'\widetilde{x} + i\widetilde{\gamma^3}p_0 + i\widetilde{\gamma^0}p_3\right)^2 = \\ &e^{-\frac{1}{\mu B'}(\widetilde{\gamma^0}p_3 + \widetilde{\gamma^3}p_0)p_1} e^{ix_*p_1}(p_1^2 + p_2^2 - (\mu B'x)^2) e^{\frac{1}{\mu B'}\left(\widetilde{\gamma^0}p_3 + \widetilde{\gamma^3}p_0\right)p_1} e^{-ix_*p_1} \end{aligned} \tag{2.14}$$

to disentangle the Dirac algebra from the x-dependence.

[1] The interaction part in the case of the minimal coupling is $(\not{p} + e\not{A})^2 = p^2 + e^2A^2 + \frac{e}{2}g^{\mu\nu}\{p_\mu, A_\nu\} + \frac{e}{2}[\gamma^\mu, \gamma^\nu][p_\mu, A_\nu]$. Because $[p_\mu, A_\nu]$ =constant for a constant field configuration, the fourth term with Dirac matrices commutes with the other terms [12]

JHEP03(2006)078

The second term in eq. (2.7) is easily calculated to be

$$v^{(0)} \equiv \mathrm{tr}\langle x|e^{isp^2}|x\rangle = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} e^{is(p_0^2-p_1^2-p_2^2-p_3^2)}dp^4 = -\frac{i}{4\pi^2 s^2}. \tag{2.15}$$

Inserting a complete set of momentum eigenstates and using eq. (2.14), the first term of eq. (2.7) can be written by

$$\begin{aligned} v^{(A)} &\equiv \mathrm{tr}\langle x|e^{-is\{p_1^2+p_2^2-(\mu B'\widetilde{x}+i\widetilde{\gamma^3}p_0+i\widetilde{\gamma^0}p_3)^2+\mu B'\gamma^2\}}|x\rangle \\ &= \frac{1}{(2\pi)^4}\left(\frac{\pi}{is}\right)^{\frac{1}{2}} \mathrm{tr}\int dp_1 dp_1' dp_0 dp_3 e^{i(p_1-p_1')\widetilde{x}}\langle p_1|e^{-is(p_1^2-\mu^2 B'^2 x^2)}|p_1'\rangle \times \\ &\quad e^{\frac{1}{\mu B'}(\widetilde{\gamma^0}p_3+\widetilde{\gamma^3}p_0)(p_1'-p_1)}e^{-is\mu B'\gamma^2}, \end{aligned} \tag{2.16}$$

where $\widetilde{x}$ is a c-number, but x in the matrix element is still an operator. Unlike the case in 2+1 dimension [10], the p_0 integration does not give a delta function of $(p_1'-p_1)$ due to the presence of p_3 momentum mixed with the Dirac algebra. Thus we have to calculate the non-diagonal matrix elements in momentum space in eq. (2.16).

Using the properties of gamma matrices

$$\left\{\widetilde{\gamma^0},\widetilde{\gamma^3}\right\} = 0, \qquad \mathrm{tr}\left(\widetilde{\gamma^3}\gamma^2\right) = 0 = \mathrm{tr}\left(\widetilde{\gamma^0}\gamma^2\right), \tag{2.17}$$

we get

$$tre^{\frac{1}{\mu B'}(\widetilde{\gamma^0}p_3+\widetilde{\gamma^3}p_0)(p_1'-p_1)}e^{-is\mu B'\gamma^2} = 4\cosh(s\mu B')\cos\left\{\frac{(p_1'-p_1)}{\mu B'}(p_0^2-p_3^2)^{1/2}\right\}. \tag{2.18}$$

Thus, $v^{(A)}$ can be written as follows

$$\begin{aligned} v^{(A)} &= \frac{4}{(2\pi)^4}\left(\frac{\pi}{is}\right)^{\frac{1}{2}}\cosh(s\mu B')\int dp_1 dp_1' dp_0 dp_3 e^{i(p_1-p_1')\widetilde{x}} \times \\ &\quad \cos\left\{\frac{(p_1-p_1')}{\mu B'}(p_0^2-p_3^2)^{1/2}\right\}\langle p_1|e^{-is(p_1^2-\mu^2 B'^2 x^2)}|p_1'\rangle. \end{aligned} \tag{2.19}$$

The matrix elements in momentum space in eq. (2.19) correspond to the matrix elements of the evolution operator for the simple harmonic oscillator with an imaginary frequency, $\omega = 2i\mu B'$ and $m = \frac{1}{2}$. Then we get

$$\langle p_1|e^{-is(p_1^2-\mu^2 B'^2 x^2)}|p_1'\rangle = \int dx' dx''\langle p_1|x''\rangle U(x'',s;x',0)\langle x'|p_1'\rangle, \tag{2.20}$$

where $U(x,s;x',0)$ is given by

$$U(x,s;x',0) \equiv \langle x,s|e^{-is(p_1^2+\frac{1}{2}\omega^2x^2)}|x',0\rangle = \left(\frac{\omega}{4\pi i\sin\omega s}\right)^{1/2} e^{i\omega\frac{(x^2+x'^2)\cos\omega s-2xx'}{4\sin\omega s}}. \tag{2.21}$$

Performing the x' and x'' integration explicitly, we get

$$\langle p_1|e^{-is(p_1^2-\mu^2 B'^2 x^2)}|p_1'\rangle = \left(\frac{i\alpha}{\pi}\right)^{\frac{1}{2}}\frac{1}{2\alpha\sin\omega s}e^{-\frac{i(p_1-p_1')^2}{8\alpha(1-\cos\omega s)}}e^{\frac{i(p_1+p_1')^2}{8\alpha(1+\cos\omega s)}}, \tag{2.22}$$

where $\alpha = -\frac{\omega}{4\sin\omega s}$.

JHEP03(2006)078

Inserting eq. (2.22) into eq. (2.19), the integration over p_1 and p_1' gives

$$v^{(A)} = \frac{2}{(2\pi)^4}\left(\frac{\pi}{is}\right)^{\frac{1}{2}}\coth(s\mu B')\mathcal{K}, \tag{2.23}$$

where, defining γ, a and p_- as follows

$$\gamma \equiv \frac{\coth(s\mu B')}{4\mu B'}, \quad a \equiv \frac{1}{\mu B'}(p_0^2 - p_3^2)^{\frac{1}{2}}, \quad p_- = p_1 - p_1', \tag{2.24}$$

$\mathcal{K}$ is given by

$$\begin{aligned}
\mathcal{K} &\equiv \frac{1}{2}\int_{-\infty}^{\infty} dp_- dp_0 dp_3 e^{i\tilde{x}p_-}(e^{iap_-} + e^{-iap_-})e^{-i\gamma p_-^2}, \\
&= \frac{1}{2}\left(\frac{\pi}{i\gamma}\right)^{1/2}\int dp_0 dp_3\left(e^{i\frac{(a+\tilde{x})^2}{4\gamma}} + e^{i\frac{(a-\tilde{x})^2}{4\gamma}}\right) \\
&= 4\pi(\mu B')^2\left(\frac{\pi\gamma}{i}\right)^{\frac{1}{2}} - 2(\mu B)^2 i\pi(\frac{\pi}{i\gamma})^{\frac{1}{2}}\int_0^1 d\xi(1-\xi)e^{i\frac{\tilde{x}^2}{4\gamma}\xi^2}.
\end{aligned} \tag{2.25}$$

Thus, $v^{(A)}$ is reduced to the following

$$\begin{aligned}
v^{(A)} = -\frac{1}{4\pi^2}\Bigg[&\frac{i}{s^2}\left\{(s\mu B')\coth(s\mu B')\right\}^{\frac{3}{2}} + \\
&+ 2(\mu B)^2\frac{1}{s}\left\{(s\mu B')\coth(s\mu B')\right\}^{\frac{1}{2}}\int_0^1 d\xi(1-\xi)e^{i\frac{\tilde{x}^2}{4\gamma}\xi^2}\Bigg].
\end{aligned} \tag{2.26}$$

Compared to the results with 2+1 dimensional Pauli term, the exponent of the first term, 3/2, is different from 1 and there appears a new term, the second term, which depends on not only the gradient but also the strength of the magnetic field.

Now we get the effective potential $V_{\rm eff}$ as follows

$$V_{\rm eff} = \frac{i}{2}\int_0^{\infty}\frac{ds}{s}e^{-ism^2}(v^{(A)} - v^{(0)}). \tag{2.27}$$

The effective potential for the uniform field configuration can be obtained by putting $\mu B' = 0$ to get

$$V_{\rm eff} = -\frac{(\mu B)^2}{4\pi^2}\int_0^{\infty}\frac{ds}{s^2}\left\{i\int_0^1 d\xi(1-\xi)e^{i(\mu B)^2\xi^2 s} - \frac{i}{2} + \frac{(\mu B)^2 s}{12}\right\}e^{-im^2 s}. \tag{2.28}$$

The divergent contributions at $s = 0$ are removed by adding local counter terms of $(\mu B)^2$, and $(\mu B)^4$. This implies the renormalization of the magnetic moment μ to the measured value and the coupling of $(\mu B)^4$ to zero presumably. The effective potential, eq. (2.28), for uniform magnetic fields is found to be real, which implies a stable magnetic background of $w = 0$. For magnetic fields weaker than the critical field $B_c = m/\mu$, using a contour integration in the fourth quadrant, the integration can be done along the negative imaginary axis giving the finite real effective action as

$$V_{\rm eff} = \frac{(\mu B)^2}{4\pi^2}\int_0^{\infty}\frac{ds}{s^2}\left\{\frac{1}{2} + \frac{(\mu B)^2 s}{12} - \int_0^1 d\xi(1-\xi)e^{(\mu B)^2\xi^2 s}\right\}e^{-m^2 s}. \tag{2.29}$$

JHEP03(2006)078

The leading radiative correction term for a weak B field is

$$\delta V_{\rm eff} = \frac{(\mu B)^6}{240\pi^2 m^2}. \tag{2.30}$$

For an inhomogeneous field configuration, $\mu B' \neq 0$, the effective potential is given by

$$\begin{aligned} V_{\rm eff} \; = \; & -\frac{(\mu B)^2}{4\pi^2}\int_0^\infty \frac{ds}{s^2}\Big\{ i\sqrt{\mu B' s\coth(\mu B' s)} \\ & \qquad \times \int_0^1 d\xi (1-\xi) e^{i\frac{(\mu B)^2}{\mu B'}\xi^2\tanh(\mu B' s)} - \frac{i}{2} + \frac{(\mu B)^2 s}{12}\Big\} e^{-im^2 s} \\ & + \frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3}\{(\mu B' s\coth(\mu B' s))^{3/2} - 1 - \frac{(\mu B' s)^2}{2}\} e^{-im^2 s}, \end{aligned} \tag{2.31}$$

where an additional divergent contribution at $s = 0$ is removed by adding a local counter term of $(\mu B')^2$ in the second term.

The leading radiative correction terms of the effective potential for a small gradient weak B field are calculated as given by

$$\delta V_{\rm eff} = \frac{(\mu B)^6}{240\pi^2 m^2} + \frac{(\mu B)^4(\mu B')^2}{288\pi^2 m^4} - \frac{(\mu B)^2(\mu B')^2}{48\pi^2 m^2} - \frac{(\mu B')^4}{960\pi^2 m^4}. \tag{2.32}$$

It is found that the effective potential, eq. (2.31), has a non-vanishing imaginary part, which implies that the background of inhomogeneous magnetic field configuration is unstable against the creation of neutral fermions with Pauli interaction. From the imaginary part of the effective potential eq. (2.31), we obtain the production rate density, $w(x) = 2Im(V_{\rm eff}(A[x]))$.

Introducing dimensionless parameters defined as $v = s\mu B'$, $\lambda = \frac{m^2}{|\mu B'|}$, $\kappa = \frac{m^2}{(\mu B)^2}$, the production rate density $w(x)$ in the unit of the fermion mass is finally given by

$$\begin{aligned} w(x) = & -\frac{2m^4}{4\pi^2\lambda\kappa}\int_0^\infty \frac{dv}{v^2}\left\{\sqrt{v\coth v}\,F\left(\frac{\lambda}{\kappa}\tanh v, \lambda v\right) - \frac{1}{2}\cos\lambda v - \frac{\lambda v}{12\kappa}\sin\lambda v\right\} \\ & -\frac{m^4}{4\pi^2\lambda^2}\int_0^\infty \frac{dv}{v^3}\left\{(v\coth v)^{3/2} - 1 - \frac{v^2}{2}\right\}\sin\lambda v, \end{aligned} \tag{2.33}$$

where

$$\begin{aligned} F(a,b) \equiv & \int_0^1 d\xi (1-\xi)\cos(a\xi^2 - b) \\ = & -\frac{1}{2a}\{\sin(a-b) + \sin(b)\} \\ & + \sqrt{\frac{\pi}{2a}}\left\{\cos(b)\,\text{FresnelC}\left(\sqrt{\frac{2a}{\pi}}\right) + \sin(b)\,\text{FresnelS}\left(\sqrt{\frac{2a}{\pi}}\right)\right\}. \end{aligned} \tag{2.34}$$

Since the scale of inhomogeneity less than Compton wavelength of the fermion is irrelevant to the particle production through this process, we take in this work the spatial gradient of the magnetic field $|B'|$ to be smaller than the ratio of field strength, $|B|$, to the Compton wavelength $\frac{1}{m}$, that is, $\lambda > \sqrt{\kappa}$.

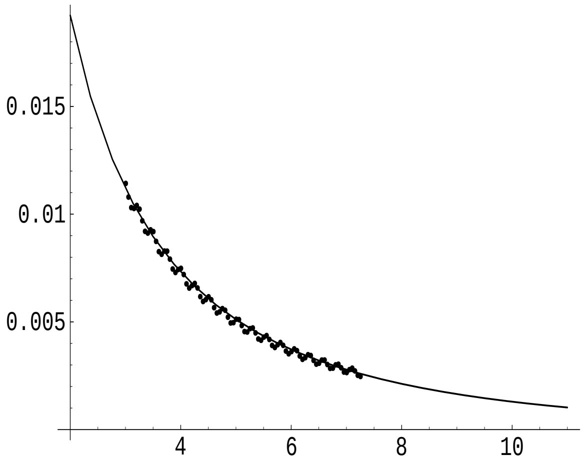

Figure 1: $\kappa = 1.0$ with varying λ: $a_{1.0} = 0.050,\ b_{1.0} = 0.136$.

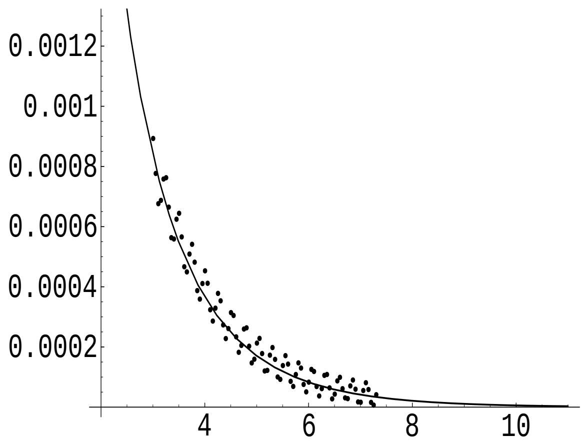

Figure 2: $\kappa = 2.0$ with varying λ: $a_{2.0} = 0.013,\ b_{2.0} = 0.775$.

The integration eq. (2.33) is a finite integration, but has singularities along the imaginary v axis similarly to the Schwinger's result, where the residue calculation gives the analytic WKB type expression. However, the integration eq. (2.33) has essential sigularities along the imaginary axis, so that it seems not possible to get a usual analytic WKB type expression using a contour integration. Therefore we use numerical integrations to investigate the properties of the production rate density given by eq. (2.33). For the numerical calculation, we consider the case of $\kappa \geq 1$ and $\lambda > 2$ as an example. Numerical integrations of the production rate density show that the second term of eq. (2.33) is negligible compared to the first term for $\lambda > 2$. The production rate shows exponential monotonic decrease for $\lambda > 2\sqrt{\kappa}$ and $\kappa \geq 1$. The production rate, $w(x)$, is calculated for $\kappa = 1.0$ and 2.0 as a function of λ. The results in the unit of $\frac{m^4}{4\pi^2}$ are as shown in figure 1 and 2. The results of numerical calculations are represented by dots in the figures. The numerical integrations of eq. (2.33) suffer from large oscillatory fluctuation, which is an unavoidable feature due to the violent oscillations in the integrand as discussed in [13]. To get an analytic expression, we obtain the best fit of the numerical results to the curves in the form of $\frac{a_\kappa}{\lambda}e^{-b_\kappa\lambda}$ for figure 1 and 2. We can observe that the particle creation rate is an exponentially decreasing function with respect to the inverse of the field gradient,

$$w \sim e^{-\text{constant}\times m^2/|\mu B'|}, \tag{2.35}$$

which shows the characteristics of the non-perturbative process. This can be understood as a quantum tunnelling through a potential barrier of height $\sim 2m$ of a particle exposed to an potential energy $\sim \mu|B'|x$ due to the inhomogeneous magnetic field coupled to the magnetic dipole moment through Pauli interaction. It is similar to the Schwinger process of electron-positron pair creation in the strong electric field, where the creation rate is decreasing exponentially [12], $w \sim e^{-\text{constant}\times m^2/|eE|}$.

One can see that the production rate is suppressed very rapidly when the field strength becomes weaker than $\sim m/\mu$ as well as the inhomogeneity scale is bigger than Compton wavelength scale. For the inhomogeneity in the Compton wavelength scale, the rate per unit time per unit volume is typically of order $\frac{m^4}{4\pi^2}$ for the critical field strength.

However for the realistic estimation of the production rate, more precise information on the mass and magnetic moment of a particle and the strength of the magnetic field as well as the scale of the spatial inhomogeneity of the field in consideration are needed for the observational possibility. As a possible environment, let us consider the pair creation of neutrinos with a non-zero magnetic dipole moment in the vicinity of the very strongly magnetized compact objects with $B = 10^{15}$G as a typical strength [14, 15]. Taking the possible magnetic moment to be as large as the experimental upper bound $\mu_\nu = 10^{-11}\mu_B$ and the mass of the neutrino to be $m_\nu \sim 10^{-2}$ eV constrained by the solar neutrino observations, the critical field strength is estimated to be $B_c \sim 10^{17}$ G. One can see that the condition for $\kappa \geq 1$ or $B < B_c$ assumed in this work is satisfied. Since the the scale of the inhomogeneity is naturally about the size of the compact object, $R \sim 10^4$ m, which is much larger than the Compton wavelength $\sim 10^{-4}$m, the production rate is expected to be substantially suppressed from the typical rate such that it may not provide sufficiently high luminosity for the neutrino detectors.

3. Discussion

We have examined the vacuum production of neutral fermions in inhomogeneous magnetic fields through Pauli interaction. The fermions, which are coupled to the background electromagnetic field through Pauli interaction, are integrated out and there appears an imaginary part in the effective action. It turns out that the production rate density depends on both the gradient and the strength of magnetic fields in 3+1 dimension, which is quite different from the result in 2+1 dimension [10], where the production rate depends only on the gradient of the magnetic fields, not on the strength of the magnetic fields. The difference can be attributed to the different nature of spinors in 3+1 and in 2+1 dimensions.

The vacuum production of fermion with the Pauli interaction is found to be a magnetic effect. Explicit calculations with a linear electric field configuration of $\hat{x}-$direction with a constant gradient along $\hat{x}-$direction such that $E_x = E_0 + E'x$ shows that the effective potential has no imaginary part when the singularities are regularized properly. It can be also shown by substituting $B \to iE$ and $B' \to iE'$ in the effective potential for a pure magnetic field eq. (2.27) with the s integration along the imaginary axis. Therefore one can see that the pair creation through Pauli interaction is a purely magnetic effect. It is an interesting result when compared to the pair creation of charged particles through the minimal coupling, which is known to be an electric effect [12].

Although the production rate density in this work has been derived for $\mu B' =$ constant, it can be applicable to various types of magnetic fields provided that the magnetic field is linear in the scale of Compton wavelength of the particle considered because the particle production rate density is a local quantity. It may be therefore applicable to a spatially slowly varying $\mu B'(x)$ as a good approximation if the gradient variation is very small in the Compton wavelength scale. For the realistic calculation of the production rate, more precise information on the mass and magnetic moment of a particle and the strength of the magnetic field as well as the scale of the spatial inhomogeneity of the field in consideration are needed for the observational possibility.

Acknowledgments

This work was supported by Korea Research Foundation Grant(KRF-2004-041-C00085).

References

[1] W. Pauli, *Relativistic field theories of elementary particles*, *Rev. Mod. Phys.* **13** (1941) 203.

[2] P.M. Lavrov, *On the effective lagrangian of QED with anomalous moments of the electron*, *J. Phys.* **A 18** (1985) 3455;
W. Dittrich and M. Reuter, *Lect. Notes Phys.* 220 (1985) 1.

[3] See, for example, C.L. Ho and P. Roy, *Quasi-exact solvability of Dirac-Pauli equation and generalized Dirac oscillators*, *Ann. Phys. (NY)* **312** (2004) 161 [hep-th/0312130];
Q.G. Lin, *Bound states of neutral particles in external electric fields*, *Phys. Rev.* **A 61** (2000) 022101 [quant-ph/0001031].

[4] K. Fujikawa and R. Shrock, *The magnetic moment of a massive neutrino and neutrino spin rotation*, *Phys. Rev. Lett.* **45** (1980) 963.

[5] For a recent review see, e.g., J.W.F. Valle, *Neutrino oscillations and new physics*, *Nucl. Phys.* **145** *(Proc. Suppl.)* (2005) 141 [hep-ph/0508067].

[6] E.K. Akhmedov, *Resonant amplification of neutrino spin rotation in matter and the solar-neutrino problem*, *Phys. Lett.* **B 213** (1988) 64;
C.S. Lim and W.J. Marciano, *Resonant spin-flavor precession of solar and supernova neutrinos*, *Phys. Rev.* **D 37** (1988) 1368;
A.B. Balantekin and C. Volpe, *Does the neutrino magnetic moment have an impact on solar physics?*, *Phys. Rev.* **D 72** (2005) 033008 [hep-ph/0411148].

[7] A. Goyal, *Photon neutrino interactions in magnetic field through neutrino magnetic moment*, *Phys. Rev.* **D 64** (2001) 013005 [hep-ph/0101313].

[8] J. Schwinger, *On gauge invariance and vacuum polarization*, *Phys. Rev.* **82** (1951) 664.

[9] G.V. Dunne and T.M. Hall, *An exact QED(3+1) effective action*, *Phys. Lett.* **B 419** (1998) 322 [hep-th/9710062]; *Borel summation of the derivative expansion and effective actions*, *Phys. Rev.* **D 60** (1999) 065002 [hep-th/9902064].

[10] Q.G. Lin, *Pair creation of neutral particles in a vacuum by external electromagnetic fields in (2+1) dimensions*, *J. Phys. G.* 25 (1999) 1793 [hep-th/9909045].

[11] G.C. McLaughlin and J.N. Ng, *Astrophysical implications of the induced neutrino magnetic moment from large extra dimensions*, *Phys. Lett.* **B 470** (1999) 157 [hep-ph/9909558].

[12] C. Itzykson and J.B. Zuber, *Quantum field theory*, McGraw-Hill, New York, 1980.

[13] J.A. Grifols, E. Massó and S. Mohanty, *Production of light pseudoscalars in external electro-magnetic fields by the Schwinger mechanism*, *Phys. Rev.* **D 65** (2002) 055004 [hep-ph/9912485].

[14] A. K. Harding, *Emission from rotation-powered pulsars and magnetars*, *AIP Conf. Proc.* 801 (2006) 241 [astro-ph/0510134].

[15] R.C. Duncan and C. Thompson, *Formation of very strongly magnetized neutron stars - implications for gamma-ray bursts*, *Astrophys. J.* 392 (1992) L9.